FORCED CONVECTION HEAT TRANSFER BEHAVIOURS OF NEWTONIAN FLUID IN A CHANNEL BETWEEN TWO PARALLEL HEATED PLATES

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ABSTRACT

Laminar forced convection heat transfer in a Newtonian fluid flow in a channel between two parallel plates has been investigated analytically. Fully developed laminar velocity distributions obtained by variable separable method was used, and viscous dissipation was taken into account. The theoretical analysis of the heat transfer is performed for three different cases of constant heat boundary conditions. The significant effect of the viscous dissipation as compared to other terms in the energy equation is manifested by the Brinkman number. In other to have a generalized idea about the viscous heating effect on the heat transfer analysis, different definitions of the Brinkman number have been used in the present study. Here, focus is on the viscous dissipative effect due to internal fluid friction for the flow of a Newtonian fluid with constant properties. The prominent role of the viscous dissipation on heat transfer characteristics has been discussed in detail for the problem under consideration subjected to different thermal boundary conditions.

Key words: Forced convection, heat transfer, Newtonian fluid, viscous dissipation, laminar flow, Parallel plates.

List of Symbols

a_1	Parameter defined in Eq. (8).	Br	Brinkman number
a_2	Parameter defined in Eqs. (30a), (30b) and (30c)	Brq_1	Modified Brinkman number
a_3	Parameter defined in Eqs. (47a), (47b) and (47c)	K	Thermal Conductivity (W/mk)
C_p	Specific Heat at Constant Pressure (J/gk)	L	width of Plate (m)
h	Convective heat transfer Coefficient	Nu	Nusselt number
P	Pressure	u	velocity (m/s)
q_1	Upper wall heat-flux (w/m_2)	q_2	Lower Wall heat-flux (w/m^2)
T	Temperature (k)	T_1	Upper wall temperature (k)
T_2	Lower wall temperature (k)	ΔT	General temperature difference (k)
U	Dimensionless velocity	w	half – channel height (m)
W	Channel height (=2w) (m)	x	Coordinate in the axial direction (m)
y	Coordinate in the vertical direction (m)	Y	Dimensionless vertical coordinate

Greek symbols

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α	Thermal diffusivity (m^2/s)	μ	dynamic viscosity (Pas)
θ	Dimensionless Temperature	θ_m	Mean dimensionless temperature
ρ	density (kg/m^2)		

Subscripts

c	centreline	m	mean

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1. Introduction

Increasing degree of miniaturization in devices, has lead to studies in micro-scale heat and fluid flow [1,2], since the thermal behavior in small devices and in micro-channels may deviate substantially from that in large objects. One of the effects that could play an important role in micro-channel is the viscous dissipation effect, and the role of the Brinkman number has been shown to be of relevance [3]. Although there have been many studies on viscous dissipation reported in the literature [4-16], it is of interest to obtain analytical results where ever possible for bench-marking and for better understanding of the process. Expressions for the Nusselt number as a function of the Brinkman number is of such basic importance, but survey showed that even for the simple flow conditions in simple geometry, the results haves not been exhaustibly reported (eg. [17-19]).

Therefore, the objectives of this study is to mathematically solve the forced convection heat transfer problem between two stationary plates subjected to different constant heat fluxes for fully developed region, which is an extension of a study by Aydin and Avci [19] where analytical expressions for Nusselt number for fully developed flow between parallel plates were reported. Our studies examined systematically, the solutions for the simple constant heat flux boundary conditions, and come to the conclusion that some of the reported results were different from what we have obtained independently. For ease of comparison, we have also followed [19] in the use of two definitions of the Brinkman number; one in terms of a temperature difference and the other in terms of a constant heat flux. The temperature distributions are also reported.

2. Mathematical Model and Formulation

Figure 1 shows the physical model and coordinate systems. A Newtonian fluid with fully developed velocity profile $u(y)$ flows between two rectangular stationary plates of $2w$ apart. The plates are convectively heated or cooled by the surrounding medium of constant heat fluxes q_1 and q_2.

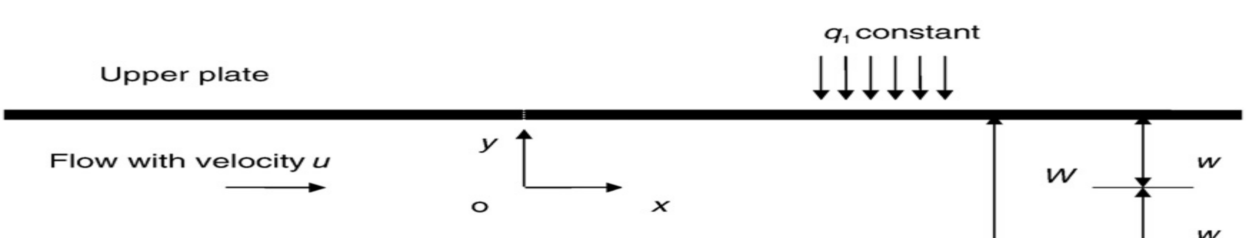

Fig. 1. Notation to the problem.

2.1 Governing Equations

The momentum and the energy equation for incompressible fluid flow are found to be relevant to this study. They are as follows:

The momentum equation is given by

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{1}$$

The velocity boundary conditions are $u=0$ when $y=w$, and $u=0$ when $y=-w$

The maximum velocity (u_c) occurs midway $(y=0)$ between the two parallel plates. Thus the well known parabolic velocity distribution is given by;

$$\frac{u}{u_c} = 1-\left(\frac{y}{w}\right)^2 \tag{2}$$

In order to illustrate the solution technique without complicating analytical procedure further, a number of simplifying assumptions are made for the simplified basic equations as:

1. The flow mode is laminar, steady and axial symmetry.
2. The fluid physical properties are independent of temperature and pressure.
3. The axial heat conduction is negligible relative to vertical heat conduction.
4. The natural convection effects are neglected.

In this case, the steady state heat balance taking viscous dissipation into account is expressed as follows:

$$\rho C_p u\frac{\partial T}{\partial x} = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Where ρ , c_p and k are the density, specific heat and thermal conductivity, respectively. In addition, the second term on the right-hand side is the viscous dissipation term effects.

2.2 Both Plates at Different Constant Heat Fluxes

Consider the case where the upper plate at constant heat flux q_1 and he lower plate at constant heat flux q_2 as shown in Fig. 1.

Substituting Eq.(2) into Eq.(3),

$$\rho c_p u_c \left(1 - \frac{y^2}{w^2}\right) \frac{\partial T}{\partial x} = K \frac{\partial^2 T}{\partial y^2} + \mu \frac{4u_c^2 y^2}{w^4} \tag{4}$$

Taking $\alpha = \frac{k}{\rho c_p}$ and for the constant heat flux case, $\frac{\partial T}{\partial x} = \frac{dT_1}{dx}$ where T_1 is the upper wall temperature, Eq. (4) becomes

$$\frac{\partial^2 T}{\partial y^2} = \frac{U_c}{\alpha}\frac{dT_1}{dx} - \frac{U_c}{\alpha}\frac{y^2}{w^2}\frac{dT_1}{dx} - \frac{\mu}{\alpha \rho C_p}\frac{4U_c^2 y^2}{w^4} \tag{5}$$

By introducing the non-dimensional quantities

$$Y = \frac{y}{w}, \quad and \quad \theta = \frac{T - T_1}{\frac{q_1 w}{k}} \tag{6}$$

Eq. (5) can be written as

$$\frac{\partial^2 \theta}{\partial Y^2} = \frac{u_c}{\alpha}\frac{wk}{q_1}\frac{dT_1}{dx} - \frac{u_c}{\alpha}\frac{4}{w^2}\frac{wk}{q_1}\frac{dT_1}{dx} w^2 Y^2 - \frac{\mu}{\alpha \rho c_p}\frac{wk}{q_1}\frac{4u_c^2 16}{w^4} w^2 Y^2 \tag{7}$$

Let $a_1 = \frac{u_c k w}{\alpha q_1}\frac{dT_1}{dx}$ and modified Brinkman number

$$Brq_1 = \frac{\mu u_c^2}{w q_1} \tag{8}$$

Therefore Eq. (7) becomes

$$\frac{d^2 \theta}{dY^2} = a_1(1 - 4Y^2) - 64 Brq_1 Y^2 \tag{9}$$

The thermal boundary conditions are

(I) $\left(-k\frac{\partial T}{\partial y}\right)=-q_1$ at $y=w$, or $\frac{\partial\theta}{\partial Y}=1$ at $Y=\frac{1}{2}$ (10a)

(II) $T=T_1$ at $y=w$, or $\theta=0$ at $Y=\frac{1}{2}$ (10b)

(III) $\left(-k\frac{\partial T}{\partial y}\right)=q_2$ at $y=-w$, or $\frac{\partial\theta}{\partial Y}=-\frac{q_2}{q_1}$ at $Y=-\frac{1}{2}$ (10c)

The solution of Eq. (9) under the above thermal boundary conditions is

$$\theta(Y)=-\frac{1}{3}Y^4\left(\frac{3}{2}\frac{q_2}{q_1}+\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\frac{q_2}{q_1}+\frac{3}{2}\right)Y^2+\left(\frac{1}{2}-\frac{1}{2}\frac{q_2}{q_1}\right)Y$$
$$+\frac{3}{32}\frac{q_2}{q_1}-\frac{13}{32}+B_r q_1\left(-8Y^4+4Y^2-\frac{1}{2}\right) \quad (11)$$

In fully developed flow, it is usual to utilize the mean fluid temperature, T_m, rather than the centre-line temperature when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m=\frac{\int \rho uTdA}{\int \rho udA} \quad (12)$$

Where $\int \rho u_c\left(1-\frac{y^2}{w^2}\right)dA=\frac{2}{3}\rho u_c Lw$ (13)

$$\int \rho u_c\left(1-\frac{y^2}{w_2}\right) TdA = \rho u_c Lw\left(\frac{q_1 w}{k}\right)\left(\frac{3}{35}\frac{q_2}{q_1}-\frac{26}{105}\right)$$
$$+ \rho Brq_1 u_c Lw\left(\frac{q_1 w}{k}\right)\left(\frac{-8}{35}\right).+ \rho u_c LwT_1\left(\frac{2}{3}\right).. \quad (14)$$

And L is the width of the plate. Therefore,

$$T_m=\left(\frac{q_1 w}{k}\right)\left(\frac{9}{70}\frac{q_2}{q_1}-\frac{13}{35}\right)-Brq_1\left(\frac{q_1 w}{k}\right)\left(\frac{12}{35}\right)+T_1 \quad (15)$$

The dimensionless mean temperature is given by

$$\theta_m=\frac{k}{q_1 w}(T_m-T_1), \quad or \quad (16)$$

$$\theta_m = \left(\frac{9}{70} \frac{q_2}{q_1} - \frac{13}{35} \right) - Brq_1 \left(\frac{12}{35} \right) \tag{17}$$

At this point, the convective heat transfer coefficient can be evaluated by the use of the defining equation

$$q_1 = h\left(T_1 - T_m\right) \tag{18}$$

Defining Nusselt number to be $Nu = \frac{hw}{k}$ (19)

$$= \frac{q_1}{T_1 - T_m} \frac{w}{k} = -\frac{1}{\theta_m} \text{, using Eq.(16)} \tag{20}$$

Therefore, on using Eqs (17) and (20), we have the new result

$$Nu = \frac{70}{-9\left(\frac{q_2}{q_1}\right) + 26 + 24\ Brq_1} \tag{21}$$

2.2.1 Upper Plate at Constant Heat Flux q_1 and Lower Plate Insulated

From Eq. (21), when $q_2 = 0$,

$$Nu = \frac{35}{13 + 12Brq_1} \tag{22}$$

This also a result not found in the literature

2.2.2 Upper Plate and Lower Plate At Equal Constant Heat Flux q_1

In this case, letting $q_1 = q_2$ in Eq. (21)

$$Nu = \frac{70}{17 + 24Brq_1} \tag{23}$$

This result is identical to that derived by Aydin and Avci [15]. For the case of no viscous dissipation, $Brq_1 = 0$, then $Nu = \frac{70}{17}$ which is an established result [16]

2.3 Solutions Using a Brinkman Number Based on Temperature Deference

In addition to the previous results, reference [15] also analyzed the problem using a Brinkman number based on a temperature difference ΔT :

$$Br = \frac{\mu u_c^2}{k\Delta T} \tag{24}$$

The dimensionless temperature is also redefined using a temperature difference instead of heat flux.

2.3.1 Upper and Lower Plates Both At Equal Constant Heat Flux

Consider the case where both plates have same input heat fluxes q_1 , which is Fig. 1 with $q_2 = q_1$. By symmetry, it is assumed that the plates will be at equal temperature, T_0, but which varies along the x direction.

Therefore, Eq. (5) becomes

$$\frac{\partial^2 T}{\partial y^2} = \frac{u_c}{\alpha}\frac{dT_0}{dx} - \frac{u_c}{\alpha}\frac{y^2}{w^2}\frac{dT_0}{dx} - \frac{\mu}{\alpha\rho c_p}\frac{4u_c^2 y^2}{w^4} \tag{25}$$

By introducing the following non-dimensional quantities,

$Y = \frac{y}{w}$, $\theta = \frac{T_0 - T}{T_0 - T_c}$, where T_c is the centreline temperature

(26)

Eq. (25) becomes

$$\frac{\partial^2 \theta}{\partial Y^2} = -\frac{u_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{(T_0 - T_c)} + 4\frac{u_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{(T_0 - T_c)}Y^2 + 64\frac{\mu}{\alpha\rho c_p}u_c^2 Y^2 \tag{27}$$

Let $a_2 = \frac{U_c}{\alpha}\frac{dT_0}{dx}\frac{w^2}{(T_0 - T_c)}$ (28)

Therefore Eq. (27) becomes

$$\frac{d^2\theta}{dY^2} = a_2(4Y^2 - 1) + 64\frac{\mu}{\alpha\rho C_p}\frac{U_c^2}{(T_0 - T_c)}Y^2 \tag{29}$$

The thermal boundary conditions are

(I) $\frac{\partial T}{\partial y} = 0$ at $y = 0$, or $\frac{\partial \theta}{\partial Y} = 0$ at $Y = 0$ (30a)

(II) $T = T_c$ at $y = 0$, or $\theta = 1$ at $Y = 0$ (30b)

(III) $T = T_0$ at $y = w$, or $\theta = 0$ at $Y = \frac{1}{2}$ (30c)

Now let $\Delta T = T_0 - T_c$

Then Eq. (24) becomes

$$Br = \frac{\mu}{k} \frac{u_c^2}{(T_0 - T_c)} \tag{31}$$

And Eq. (29) becomes

$$\frac{d^2\theta}{dY^2} = a_2\left(4Y^2 - 1\right) + 64BrY^2 \tag{32}$$

The solution of Eq. (32) under the above thermal boundary conditions can be shown to be

$$\theta(Y) = \left(1 - \frac{24}{5}Y^2 + \frac{16}{5}Y^4\right) - \frac{8}{5}Br\ (Y^2 - 4Y^4) \tag{33}$$

In dimensional form, the temperature distribution is

$$T = (T_0 - T_c)\left(\frac{24}{5}Y^2 - 1 - \frac{16}{5}Y^4\right) + \frac{8}{5}Br\ (T_0 - T_c)\left(Y^2 - 4Y^4\right) + T_0 \tag{34}$$

Similar to Eqs. (12) – (14), the mean temperature can be found to be

$$T_m = -\frac{136}{175}(T_0 - T_c) + \frac{8}{175}Br\ (T_0 - T_c) + T_0 \tag{35}$$

The dimensionless mean temperature is defined as

$$\theta_m = \frac{T_m - T_0}{T_c - T_0} \tag{36}$$

From which is obtained

$$\theta_m = \frac{136}{175} - \frac{8}{175}Br \tag{37}$$

Upon using

$$q_1 = h\left(T_0 - T_m\right) = k\left(\frac{\partial T}{\partial y}\right)_{y=w} \tag{38}$$

The expression for h is

$$h = \frac{k\left(\dfrac{\partial T}{\partial y}\right)_{y=w}}{T_0 - T_m} \tag{39}$$

From Eq. (26),

$$\frac{\partial T}{\partial y} = \frac{\left(T_c - T_0\right)}{w}\frac{\partial \theta}{\partial Y} \tag{40}$$

In terms of θ_m, h is given by

$$h = \frac{\dfrac{k\left(T_c - T_0\right)}{w}\left(\dfrac{\partial \theta}{\partial Y}\right)_{y=0.5}}{T_0 + \theta_m\left(T_0 - T_c\right) - T_0} . = \frac{-k\left(\dfrac{-16}{5} + \dfrac{8}{5}Br\right)}{w\left(\dfrac{136}{175} - \dfrac{8}{175}Br\right)} \tag{41}$$

or $$Nu = \frac{35(2 - Br)}{17 - Br} \tag{42}$$

In this result, when $Br = 0$, the Nusselt number also reduces to $Nu = \dfrac{70}{17}$ as before [16]. It is noted that in [15], the result Eq. (42) is not mentioned, but temperature equation corresponding to Eq. (32) is given. However, we found that our results are different. In our Eq. (32), the second term on the right hand side has the coefficient 64, but in [15], it is 16. In our Eq. (33), the coefficient in the second term is $\dfrac{8}{5}$, but in [15], it is $\dfrac{2}{5}$.

2.3.2 Upper Plate at Constant Heat Flux and Lower Plate Insulated

Consider the case where the upper plate is at constant heat flux and lower plate is insulated, which is Fig. 1 with $q_2 = 0$.

It is assumed that upper plate is kept at temperature T_1 and lower plate is kept at temperature T_2, both T_1 and T_2 varying along the $x-$direction.

By introducing the following non-dimensional quantities,

$$Y = \frac{y}{w},\ \theta = \frac{T_1 - T}{T_1 - T_2} \tag{43}$$

Eq. (5) becomes

$$\frac{\partial^2\theta}{\partial Y^2} = -\frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{w^2}{\left(T_1 - T_2\right)} + 4\frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{w^2}{\left(T_1 - T_2\right)}Y^2 + 64\frac{\mu}{\alpha\rho c_p}\frac{u_c^2}{\left(T_1 - T_2\right)} \tag{44}$$

Let $a_3 = \frac{u_c}{\alpha}\frac{dT_1}{dx}\frac{w^2}{\left(T_1 - T_2\right)}$ (45)

Then Eq. (44) becomes

$$\frac{d^2\theta}{dY^2} = a_3(4Y^2 - 1) + 64\frac{\mu}{k}\frac{U_c^2}{(T_1 - T_2)}Y^2 \tag{46}$$

The thermal boundary conditions here are

(I) $k\frac{\partial T}{\partial y} = 0$ at $y = -w$, or $\frac{\partial\theta}{\partial Y} = 0$ at $Y = -\frac{1}{2}$ (47a)

(II) $T = T_1$ at $y = w$, or $\theta = 0$ at $Y = \frac{1}{2}$ (47b)

(III) $T = T_2$ at $y = -w$, or $\theta = 1$ at $Y = -\frac{1}{2}$ (47c)

Now let $\Delta T = T_1 - T_2$ Therefore Eq. (24) becomes

$$Br = \frac{\mu}{k}\frac{u_c^2}{(T_1 - T_2)} \tag{48}$$

Then Eq. (46) becomes

$$\frac{d^2\theta}{dY^2} = a_3\left(4Y^2 - 1\right) + 64BrY^2 \tag{49}$$

The solution of Eq. (49) under the above thermal boundary conditions is

$$\theta\left(Y\right) = \left(Y^4 - \frac{3}{2}Y^2 - Y + \frac{13}{16}\right) + Br\ (8Y^4 - 4Y^2 + \frac{1}{2}) \tag{50}$$

From Eq. (43) and (50), the temperature distribution is

$$T = \left(T_2 - T_1\right)\left(Y^4 - \frac{3}{2}Y^2 - Y + \frac{13}{16}\right) + Br\left(T_2 - T_1\right)\left(8Y^4 - 4Y^2 + \frac{1}{2}\right) + T_1 \tag{51}$$

Using Eq. (12), the expression for T_m is found to be

$$T_m = (T_2 - T_1)\frac{26}{35} - (T_2 - T_1)\ Br\frac{12}{35} + T_1 \qquad (52)$$

The dimensionless mean temperature is defined as

$$\theta_m = \frac{T_m - T_1}{T_2 - T_1} \qquad (53)$$

Therefore the expression for dimensionless means temperature is

$$\theta_m = \frac{26}{35} - \frac{12}{35}\ Br \qquad (54)$$

Following Eq. (20), the convective heat transfer is given by

$$h = \frac{\dfrac{k(T_2 - T_1)}{w}\left(\dfrac{\partial\theta}{\partial Y}\right)_{y=0.5}}{-(T_2 - T_1)\left(\dfrac{26}{35}\right) + (T_2 - T_1)Br\left(\dfrac{12}{35}\right)} . = \frac{35\left(\dfrac{k}{w}\right)(-2)}{-26 + 12Br} \qquad (55)$$

or $$Nu = \frac{35}{13 - 6Br}. \qquad (56)$$

In this result when $Br = 0$, the Nusselt number also reduces to Nu = , which is identical to the result derived in section 2.2.1 when $Brq_1 = 0$.

3 Graphical Results and Discussions

The previous equations give the most useful results in the present study. This section discuses briefly the graphical plots of those equations.

3.1 Both Plates Kept at Different Constant Heat Fluxes q_1 and q_2

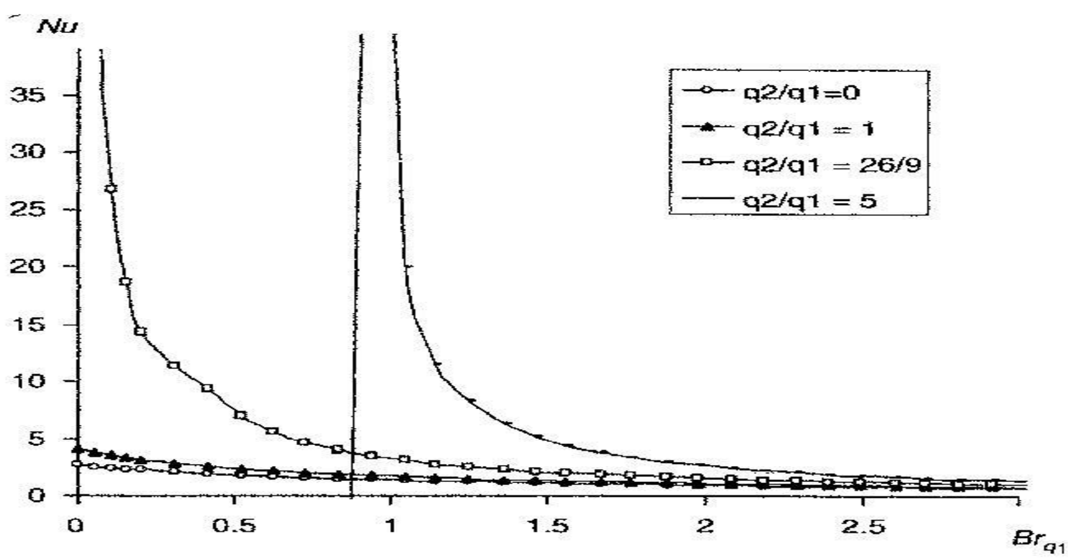

Fig. 2. Graph of Nu versus Br_{q_1}

The main physical quantity of interest, is the Nusselt number which represents the heat transfer rate and the wall of the plate. The variation of the Nusselt number with the Brinkman number needs to be investigated. To demonstrate the effect of viscous dissipation on the Nusselt number, Eq. (21) is considered. However, the variation of the Nusselt number with Br_{q_1} is shown in Fig 2 for heat flux ratio $\frac{q_2}{q_1} = 0, 1, \frac{26}{9}$ and 5. The choice of different heat flux ratios represents different cases. The ratio $\frac{q_2}{q_1} = 0$ corresponds to the case of an insulated lower plate. Similarly, 1 corresponds to the case where both plates are at equal constant heat flux. The ratio $\frac{26}{9}$ indicates the special case occurring due to the point of singularity at the origin.

One may notice from the above figure that the variation of the Nusselt number with Br_{q_1} is not continuous for all the cases considered in the study, rather a clear existence of the point of singularity is observed in each case at different point at a different Br_{q_1}, as suggested by Equation (21). The different locations of the point of singularity are due to the different ratios of heat flux considered, and at this point, the shear heating balances the heat supplied by the wall. However, from this point of singularity as Br_{q_1} increases in the positive direction $\left(Br_{q_1} > 0\right)$, the Nusselt number decreases because of the decrease in the driving potential of the heat transfer, and it finally attains different constant values asymptotically, (when $Br_{q_1} \to \infty$), for all the cases of heat flux taken into account.

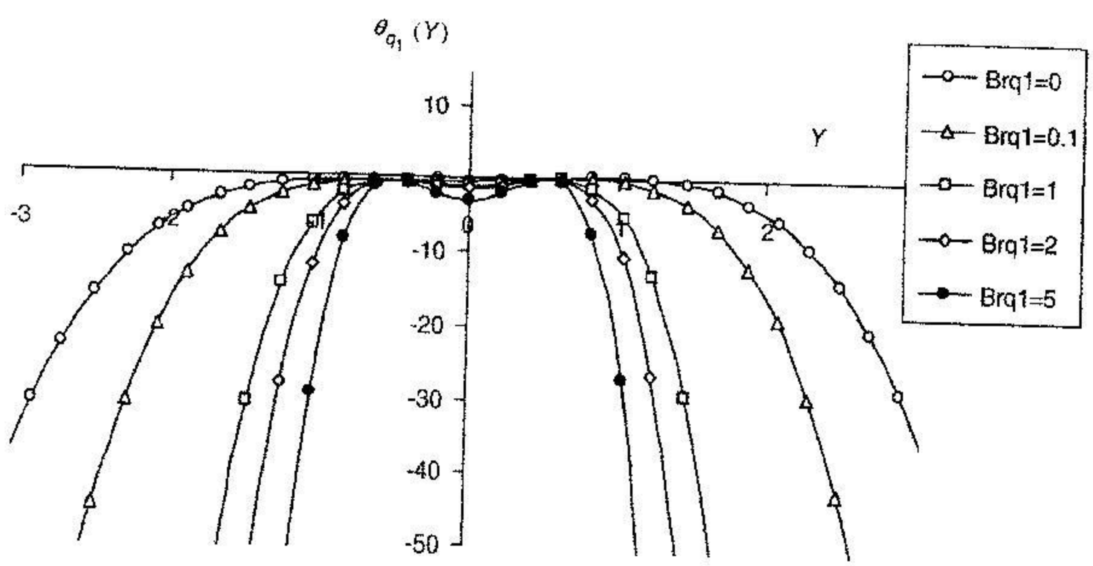

Fig .3. Graph of $\theta q_1\left(Y\right)$ versus Y for the case insulated lower plate

The Brinkman number is an important parameter governing heat transfer and fluid flow between two parallel plates. Effects of viscous dissipation in a fluid flow and heat transfer phenomenon is explained by the Brinkman number. The paper aims in finding out the influence of viscous dissipation effects on the temperature profile and the resulting Nusselt number. Fig. 3 depicts the dimensionless temperature profile within the flow field for different Brq_1**,** pertaining to the case where plates are kept at different constant heat flux conditions obtained from Equation (11). In the case of insulated lower plate $\frac{q_2}{q_1} = 0$,

Let

$$\theta q_1(Y) = -\frac{1}{2}Y^4 + \frac{3}{4}Y^2 + \frac{1}{2}Y - \frac{13}{32} + Brq_1\left(-8Y^4 + 4Y^2 - \frac{1}{2}\right)$$

One may observe that with increasing value of Brq_1, the temperature increases as expected. Positive values of Brq_1 are compactible with the wall heating case which indicates heat transfer to the fluid across the wall. Therefore, in the cases with positive Brq_1, the fluid temperature increases as evident from the above figure which shows that the curves converge at $(0.5, 0)$ and are not symmetrical about the vertical axis.

3.2 Both Plates at Equal Constant Heat Flux

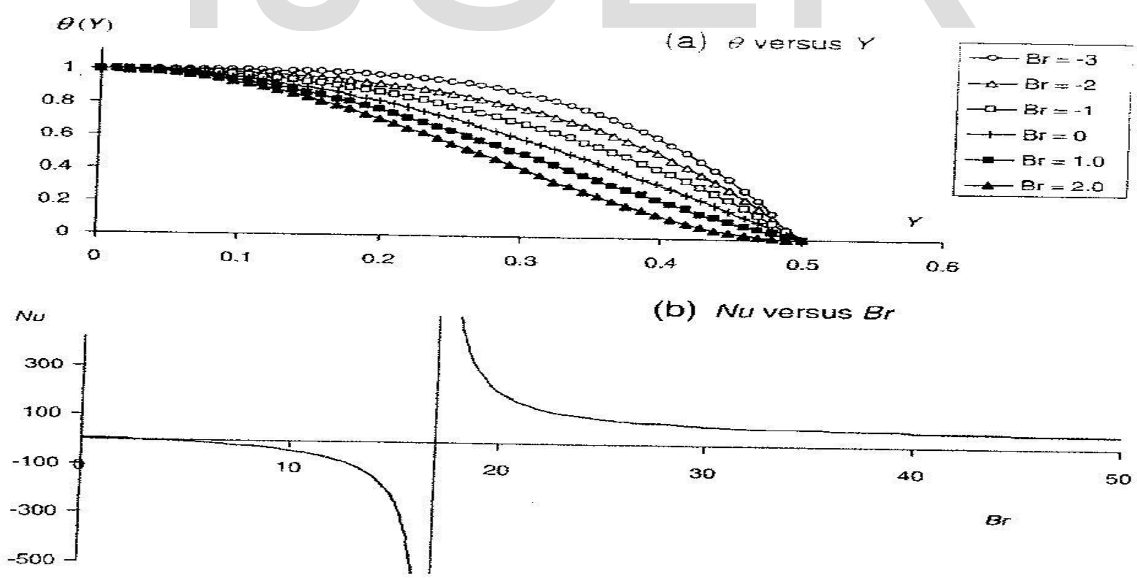

Fig. 4. variations of temperature and Nusselt number for the case of equal heat fluxes

Here, the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number defined in Equation (31), is discussed through presentation of their graphical plots obtained from Equation (33) which is shown in Fig 4(a). The coresopnding Nuselt number for this case is Eq. (42) as shown in Fig 4(b).

Viscous dissipation always generates a distribution of heat source, stimulating the internal energy in the fluid, and hence the temperature profile gets distorted as is envisaged in Figure 4(a) above. Figure 4(a) depicts the dimensionless temperature profile within the flow field for the wall heating case and wall cooling case respectively. As explained earlier that for wall heating case, the fluid temperature increases where as the reverse is true for the wall cooling case. Figure 4(b) exhibits the variation of the Nusselt number with Br. However , compared to cases with an insulated lower plate, the variation of the Nusselt number shows a distinct feature as Br changes in the case of the equal constant heat flux condition. It is important to observe that the increase in Br values decreases the Nusselt number and the asymptote appears at $Br = 17$. However, from the point of singularity the Nusselt number reaches a constant value in either direction asympiotically.

3.3 Upper Plate at Constant Heat Flux and Lower Plate Insulated

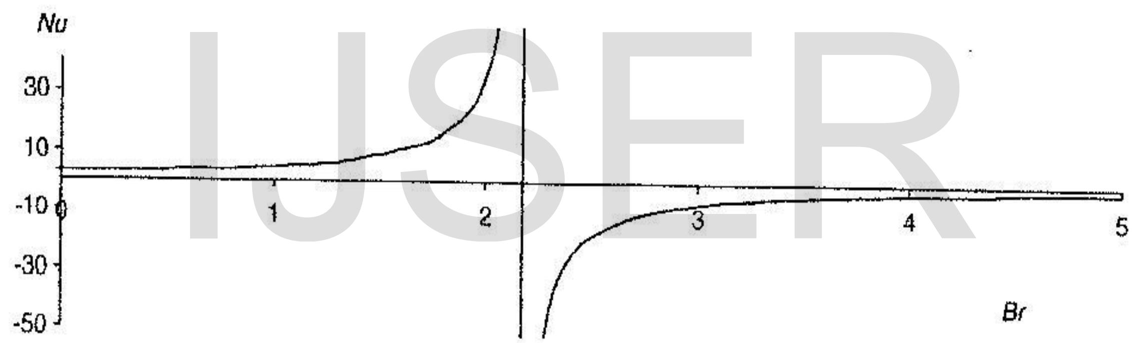

Fig. 5: Graph of Nusselt number versus Br for insulated lower plate

The variation of Nusselt number as depicted in Figure 5 shows similarity with the results of the published literature [13] for no viscous dissipation, $Nu = \frac{35}{13}$. in increasing Br makes the bulk temperature of the fluid to increase and hence the driving potential of the heat transfer is reduced, which is reflected on the variation of the Nusselt number as Br increases in the positive direction. However, the Nusselt number decreases asymptotically as $Br \to \infty$.

4. Conclusion

The forced convection heat transfer problem with viscous dissipation between two parallel plates subjected to different thermal boundary conditions has been solved mathematically, which is an extension of a study by Aydin and Avci [15]. Completely analytical solutions for the fluid temperature and resulting Nusselt number (Nu) have been derived, the effects of the Brinkman number (Br) on the temperature distribution and Nusselt number have been shown through graphical plots. The following conclusions are drawn.

1. The Nusselt number in the thermal region tends to decrease with an increase in the Brinkman number.
2. It has been shown that viscous dissipation in the fluid can significantly influence laminar flow heat transfer.
3. With regard to this study, the present analytical method can be applied to heat transfer not only in a channel between parallel plates but also in a concentric annulus.
4. It can also be applied to heat transfer in a channel with a moving wall because there is no restriction on the velocity distribution from a fluid.

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